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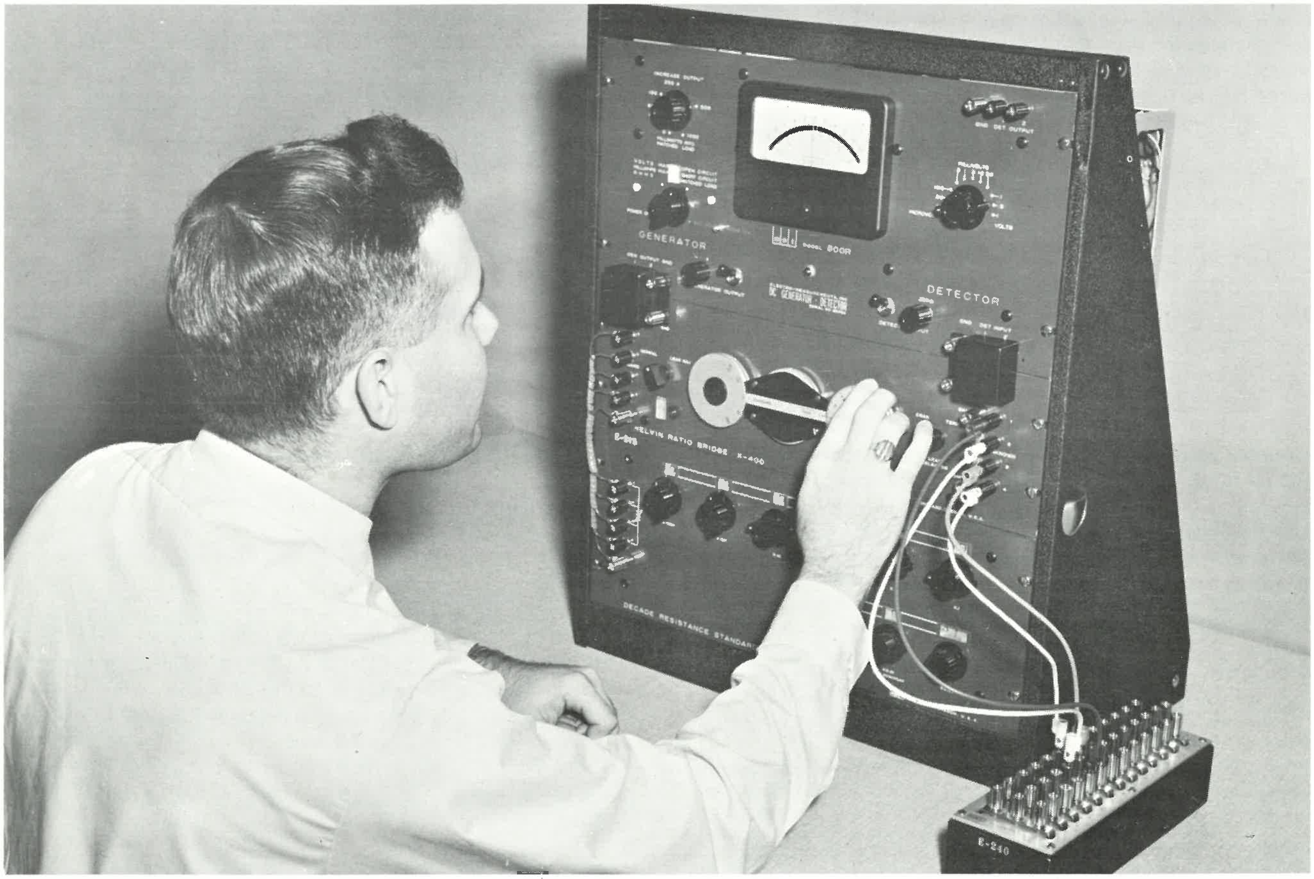
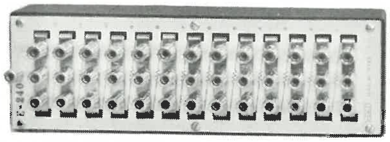


VOLTAGE DIVIDER CALIBRATION

In the last few years the measurement, calibration, and control fields have experienced an increasing need for more accurate resistive voltage dividers. Fortunately, improved materials for resistors, better construction methods, and more precise calibration techniques are all helping to satisfy this demand. As one of the pioneers in the use of these new materials, methods, and techniques, ESI is producing voltage dividers with linearity much better than 10 parts per million. A few years ago this accuracy was only obtainable by the use of complex techniques in primary standards laboratories. In the first issue of DESIGN IDEAS we will discuss the precise calibration techniques developed at ESI for measuring such extreme linearities.

To make precision linearity measurements of a voltage divider, we first calibrate a standard divider to find its deviation, then use this standard to adjust a transfer divider to less than 0.2 ppm deviation, and finally, compare the transfer divider to each decade of the production model. We could use the standard divider to calibrate the production divider directly, but the additional calculations that would be necessary, make it much more convenient to use the essentially perfect transfer divider.

Calibration of a Standard Divider



The standard divider that we are going to calibrate is a string of ten nominally equal resistors which will divide a voltage into ten nominally equal parts. The output of this divider can be set to integral multiples of a tenth of the input voltage. To calibrate this divider, we need to know the difference between the actual ratio of input to output voltages and the setting. This difference is called linearity deviation.

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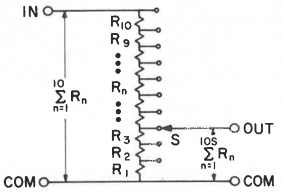
$$\Gamma \equiv \frac{E_{out}}{E_{in}} - S$$

Γ — linearity deviation
E_{in} — actual input voltage
E_{out} — actual output voltage
S — divider setting $= \dfrac{\text{nominal output voltage}}{\text{nominal input voltage}}$

The usual way to calibrate a divider is to compare it with a more accurate divider. In this case, we can't compare it with a more accurate one because this divider is to be the standard of accuracy. Therefore, we need a different calibration technique.

Since the voltage and resistance divide proportionately, the linearity deviation can be found by a precision comparison of the resistors in the divider string. The linearity deviation equation can be written in terms of the resistors (see Figure 1).



$$\Gamma = \frac{\sum_{n=1}^{10S} R_n}{\sum_{n=1}^{10} R_n} - S$$

Γ — linearity deviation in per unit
R_n — resistance of nth resistor in ohms
$\sum_{n=1}^{10S} R_n$ — resistance from COM to OUT in ohms
$\sum_{n=1}^{10} R_n$ — total input resistance in ohms
S — setting in per unit

Figure 1. Linearity deviation in terms of resistance.

If all of the resistors in the string were equal, the voltage would divide equally and the unit would be linear. Our problem is to find how different our divider is from this ideal. To do this, each resistor, R_n, of the string is compared to the standard resistor, R_s. To maintain the ultimate in measurement accuracy, precise, four-terminal resistance comparisons are made (see Figure 2).

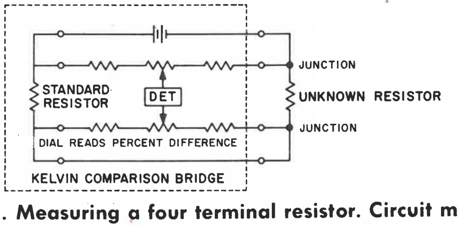


Figure 2a. Measuring a four terminal resistor. Circuit measures resistance between junctions shown.

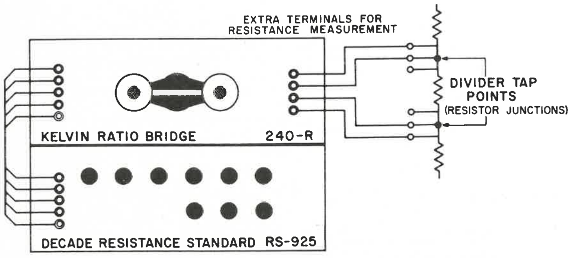

EXTRA TERMINALS FOR RESISTANCE MEASUREMENT
DIVIDER TAP POINTS
(RESISTOR JUNCTIONS)
KELVIN RATIO BRIDGE
240-R
DECADE RESISTANCE STANDARD RS-925

Figure 2b. Measuring resistance between taps of a standard divider.

The two extra terminals for this purpose are located on either side of the taps of the standard divider. The resulting measurement is of the resistance between the tap junctions. This is the resistance which is responsible for the voltage division. The measurements are made with the ESI Model 242-R Kelvin Ratio Bridge (see Figure 3), which is capable of such comparisons to better than 1 ppm.

Figure 3. ESI Model 242 Kelvin Ratio Bridge.

The measured resistance deviation $\triangle_n$ is used to calculate the linearity deviation,

$$R_n \equiv R_s (1+\Delta_n)$$

R_n — resistance of the nth resistor in ohms
R_s — resistance of the standard resistor in ohms
Δ_n — per unit deviation of the unknown from the standard

We substitute this new expression for R_n in Fig. 1 to get

$$\Gamma = \frac{\cancel{R_s}\sum_{n=1}^{10S}(1+\Delta_n)}{\cancel{R_s}\sum_{n=1}^{10}(1+\Delta_n)} - S = \frac{\sum_{n=1}^{10S}\Delta_n - S\sum_{n=1}^{10}\Delta_n}{10 + \sum_{n=1}^{10}\Delta_n}$$

The precise value of the standard resistor is unimportant because it is cancelled out in the equation. Thus voltage division accuracy is dependent only on the relative values of the resistors in the divider, and not on their absolute resistance value.

The linearity deviation equation is modified to simplify calculations. We define the average resistance deviation,

$$\Delta_{AV} \equiv \frac{1}{10}\sum_{n=1}^{10}\Delta_n$$

then the terms of the linearity deviation equation are rearranged to give,

$$\Gamma = \frac{\sum_{n=1}^{10S}(\Delta_n - \Delta_{AV})}{10(1+\Delta_{AV})}$$

Since the value of $\triangle_{AV}$ is much less than one, it can be removed from the denominator without seriously affecting the accuracy of Γ. This results in the form which is used for the actual calibrations:

$$\Gamma \approx \overbrace{\frac{1}{10}\underbrace{\sum_{n=1}^{10S}(\Delta_n - \underbrace{\Delta_{AV}}_{\text{STEP 1}})}_{\text{STEP 2}}}^{\text{STEP 3}}$$

Figure 4 is given to illustrate the method and the typical accuracy found.
The steps involved are:

Step 1. Calculate the average deviation ($\triangle_{AV}$) from the measured deviations ($\triangle_n$).

Step 2. Subtract the average from each deviation to get the difference from the average.

Step 3. Sum these differences from the average and divide by 10 to get the linearity deviations at each tap.

RESISTOR NUMBER	MEASURED DEVIATION PPM	DIFFERENCE FROM THE AVERAGE PPM	LINEARITY DEVIATION Γ PPM
10S	Δ_n	$\Delta_n - \Delta_{AV}$	$\frac{1}{10}\sum_{n=1}^{10S}(\Delta_n - \Delta_{AV})$
1	−4	−0.4	−0.04
2	0	+3.6	+0.32
3	−4	−0.4	+0.28
4	−3	+0.6	+0.34
5	−1	+2.6	+0.60
6	−5	−1.4	+0.46
7	−2	+1.6	+0.62
8	−6	−2.4	+0.38
9	−2	+1.6	+0.54
10	−9	−5.4	0
TOTAL	$\sum_{n=1}^{10}\Delta_n = -36$	0	

Figure 4. Standard divider deviations.

Adjustment of a Transfer Divider

Next the transfer divider is adjusted to a linearity deviation of less than ±0.2 ppm by comparison to corresponding settings of our calibrated standard divider. To assure the ultimate in accuracy, compensation must be made for the effects of lead resistances. (see Figure 5).

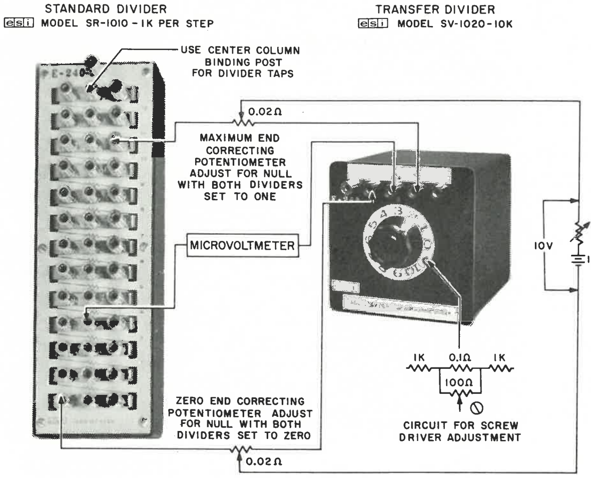

STANDARD DIVIDER
esi MODEL SR-1010 - 1K PER STEP
TRANSFER DIVIDER
esi MODEL SV-1020-10K
USE CENTER COLUMN BINDING POST FOR DIVIDER TAPS
0.02Ω
MAXIMUM END CORRECTING POTENTIOMETER ADJUST FOR NULL WITH BOTH DIVIDERS SET TO ONE
MICROVOLTMETER
10V
12V
1K
0.1Ω
1K
100Ω
ZERO END CORRECTING POTENTIOMETER ADJUST FOR NULL WITH BOTH DIVIDERS SET TO ZERO
CIRCUIT FOR SCREW DRIVER ADJUSTMENT
0.02Ω

Figure 5. Adjusting the transfer divider.

Voltage drops across the lead resistances to the dividers will cause their outputs to be a little above zero at the zero setting, and a little below the input at maximum. By adjusting the end correcting potentiometers in the comparison circuit, we can make both dividers have the same output at the zero and maximum taps. When this is done, the comparison of the dividers will be independent of the external connections.

The transfer divider has adjustable potentiometers at all settings ftrom 0.1 through 0.9. At corresponding settings of the divider these potentiometers are adjusted until the linearity deviation that we calculated for the standard divider is read on a high impedance calibrated microvoltmeter. This procedure theoretically reduces the linearity deviation of the transfer divider to zero. Careful consideration of the known possible sources of error and cross-checking with other calibration methods confirm that a recently calibrated transfer divider will have a linearity deviation of less than 0.2 ppm.

Precision Calibration of a Production Divider

We will use the transfer divider to calibrate an ESI model RV-622 DEKAVIDER® decade voltage divider. The specified linearity of the Dekavider is 10 ppm, but to leave an ample safety factor for age and temperature effects, final inspection tolerances of all ESI dividers are typically better than twice as good as their published specifications. The better than 0.2 ppm accuracy of the transfer divider permits us to verify these values with confidence.

Before we proceed with the calibration, we need to know a little about the Kelvin Varley circuit of the DEKAVIDER.

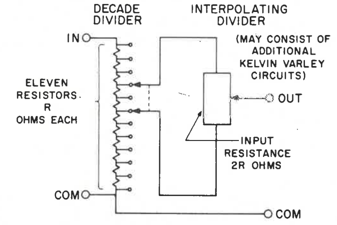

DECADE DIVIDER
INTERPOLATING DIVIDER
(MAY CONSIST OF ADDITIONAL KELVIN VARLEY CIRCUITS)
IN
ELEVEN RESISTORS R OHMS EACH
OUT
INPUT RESISTANCE 2R OHMS
COM
COM

Figure 6. Kelvin Varley divider circuit.

In Figure 6 we see that the switch for each decade actually positions an interpolating divider to cover a chosen one-tenth of the range of the decade. The one exception is the last decade, which has no interpolating divider between its steps. The dials of each decade except the last can be set to ten positions, 0 through 9. The last dial can be set to an additional position, TEN. With this additional position the output can be set equal to the input by placing the dials at 0.999,99 TEN.

The linearity deviation of a Kelvin Varley divider can be measured in steps. First the contribution of the lead and contact resistance is measured. This resistance will cause the output of the DEKAVIDER to be a little above zero at zero setting and a little below input at maximum. These small linearity deviations, about 0.3 ppm for a 10 K divider, are measured with a microvoltmeter between the common and output taps of the DEKAVIDER with the dial set at 0.000,000, and between the input and output taps with a setting of 0.999,99 TEN. For a ten kilohm divider ten volts is supplied so that the meter will read 10 microvolts per ppm. Next we measure the linearity deviation of the first decade. To do this, we first eliminate the end deviations just measured, so that they will not enter our data twice. This is accomplished by adjusting end correcting potentiometers as was done in adjusting the transfer divider. The settings of the first decade are then checked with the other dials, first at zero, then at maximum (for instance, 0.200,000 and 0.299,99 TEN). Next, the supply voltage and transfer divider are connected across the second decade (Figure 7). The end correcting potentiometers are again adjusted for

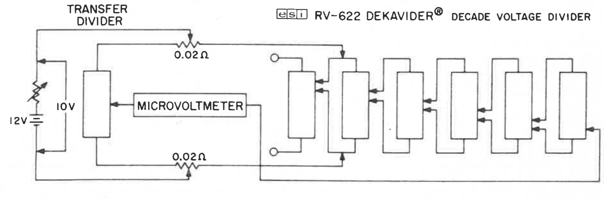


Figure 7. Connection for calibration of individual RV-622 decades.

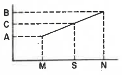
agreement at zero and full scale. Each position of this decade is then checked with the last four dials at both 0,000 and 9,99 TEN. The same measurement procedure is followed for each decade. Since the second decade covers only one-tenth of the total divider input, the linearity deviation measurements made on it by this technique must be divided by ten to determine their effect on the linearity deviation of the RV-622. The third decade measurements must be divided by 100, and so on. Typical resulting values are shown in Figure 8.

RANGE OF DECADE	FIRST DECADE	SECOND DECADE	THIRD DECADE	FOURTH DECADE	FIFTH DECADE	SIXTH DECADE
0 TO 0X*	0.0 TO +0.6	0.00 TO −0.10	0.000 TO −0.050	0.000,0 TO −0.001,0	0.000,00 TO +0.000,20	0.000,000
1 TO 1X	−0.3 TO 0.0	+0.05 TO −0.14	+0.003 TO −0.050	−0.007,0 TO −0.006,0	+0.000,50 TO +0.000,90	−0.000,140
2 TO 2X	−0.7 TO +0.4	−0.03 TO −0.10	+0.030 TO −0.020	+0.003,0 TO +0.005,0	+0.000,70 TO +0.000,90	−0.000,011
3 TO 3X	−0.5 TO +0.5	−0.07 TO −0.11	+0.060 TO +0.010	+0.004,0 TO +0.008,0	+0.000,60 TO +0.000,60	−0.000,040
4 TO 4X	+0.1 TO +1.1	+0.06 TO +0.05	+0.080 TO +0.030	+0.013,0 TO +0.012,0	+0.000,10 TO +0.000,00	−0.000,020
5 TO 5X	+0.5 TO +1.4	+0.21 TO +0.14	+0.100 TO +0.035	+0.019,0 TO +0.019,0	+0.000,30 TO +0.000,90	−0.000,060
6 TO 6X	+0.5 TO +1.0	+0.21 TO +0.09	+0.090 TO +0.010	+0.021,0 TO +0.023,0	−0.000,20 TO +0.000,40	−0.000,050
7 TO 7X	+0.4 TO +0.8	+0.20 TO +0.02	+0.080 TO 0.000	+0.020,0 TO +0.020,0	+0.000,70 TO +0.000,80	−0.000,100
8 TO 8X	0.0 TO +0.3	+0.14 TO +0.01	+0.060 TO −0.010	+0.023,0 TO +0.021,0	+0.000,10 TO +0.000,20	−0.000,080
9 TO 9X	−0.4 TO 0.0	+0.14 TO 0.00	+0.050 TO 0.000	+0.015,0 TO 0.000,0	−0.000,03 TO 0.000,00	−0.000,120 0.000,000

*X — means that the decades to the right of the one being measured are set to maximum. For example 0.4X for the second decade indicates a dial setting of 0.049,99TEN.

Figure 8. Linearity deviation measurements of a Model RV-622 DEKAVIDER® decade voltage divider.

To determine the accuracy of a particular setting from the data of Figure 8, we can interpolate between the recorded values. An example is shown in Figure 9.



$$C = A + \frac{(B-A)(S-M)}{(N-M)}$$

D — decade for which calculation is being made
S — setting of decade D and decades following
M — setting of decade D with following decades at zero
N — setting of decade D with following decades at 99••• TEN
A — linearity deviation at M
B — linearity deviation at N
C — linearity deviation at S (calculated)

D	END DEVIATION	FIRST DECADE	SECOND DECADE	THIRD DECADE	FOURTH DECADE	FIFTH DECADE	SIXTH DECADE	TOTAL
M	0.000,000	0.500,000	0.040,000	0.003,000	0.000,200	0.000,010	—	
N	0.999,99TEN	0.599,99TEN	0.049,99TEN	0.003,99TEN	0.000,29TEN	0.000,01 TEN	—	
S	0.543,212	0.543,212	0.043,212	0.003,212	0.000,212	0.000,012	0.000,002	
Appm	+0.3	+0.5	+0.06	+0.06	+0.003	+0.000,5	—	
Bppm	−0.3	+1.4	+0.05	+0.01	+0.005	+0.000,9	—	
Cppm	−0.025,930	+0.888,908	+0.056,788	+0.049,400	+0.003,240	+0.000,580	−0.000,011	+0.972,975

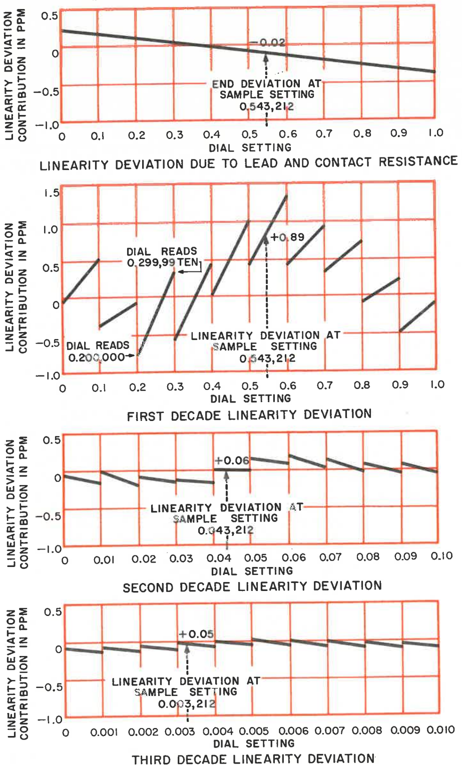
Data taken from FIGURE 10 for a DEKAVIDER® setting of 0.543,212

Figure 9. Calculating the linearity deviation at a sample setting of 0.543,212.

The contributions of the last three decades can be ignored because they are less than the uncertainty of the measurements of the first decade. For this reason, detailed data from only the first three decades is recorded on the calibration certificate supplied with the RV-622. This certificate also contains the values of the end linearity deviations, and the maximum linearity deviations of each of the last three decades.

Using the Calibration Certificate

The data from the calibration certificate can be numerically interpolated as in Figure 9 or graphically as in Figure 10 (data from Figure 8). The same sample setting that was used in Figure 9 is shown as an example. The contributions from the lead and contact resistances, and from each of the first three decades are added to give the linearity deviation at the sample setting.



TOTAL: −0.02 + 0.89 + 0.06 + 0.05 = 0.98 PPM LINEARITY DEVIATION

Figure 10. Graphical interpolation of data from calibration certificate.

Power Coefficient

For best calibration accuracy, all of our measurements have been made with low power input. What effect will enough input power to produce appreciable resistance heating have on the reliability of our data? All RV-622 dividers are checked twice—once with an input of about 20 milliwatts, and again with maximum rated input power of 5 watts. A dissipation of five watts raises the temperature of nine of the first decade resistors by 12°C. The other one-tenth of the input resistance changes only about 3°C because of the Kelvin Varley circuit used. Because of this temperature difference, it is not enough to match the temperature coefficients—the coefficients must be very low. ESI has pioneered the use of a special alloy wire for making resistors. By selecting the temperature coefficient of the wire and using special winding techniques, an almost zero temperature coefficient is maintained over a wide temperature range. The results of using such resistors in ten 10 Kilohm RV-622 DEKAVIDER® decade voltage dividers with 0.2 and 5.0 watt inputs are shown in Figure 11.

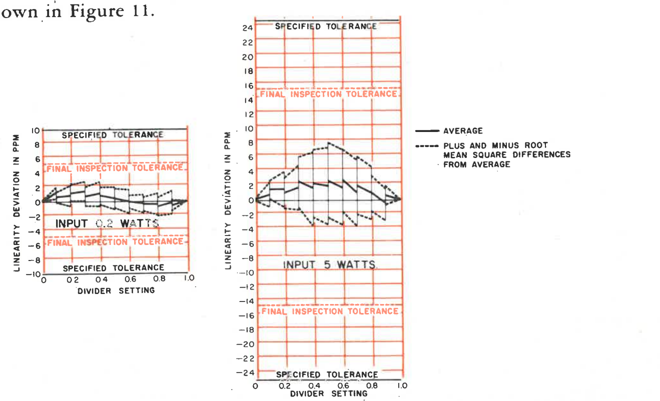


Figure 11. Linearity deviation of ten 10 K RV-622 DEKAVIDER® decade voltage dividers.

The average values of the linearity deviation were calculated for the 0.H00,000 and 0.H99,99 TEN settings of the first decade. The effects of the second and all following decades are so small that they can be neglected without materially changing the result. These curves demonstrate that the techniques employed by ESI consistently produce voltage dividers substantially better than specified tolerances. Such conservative ratings are your guarantee of measurement accuracy under prolonged use.

Litho in U.S.A. esi